

## Supplement 1-2: Types of Rates

*Silly Warm up:*

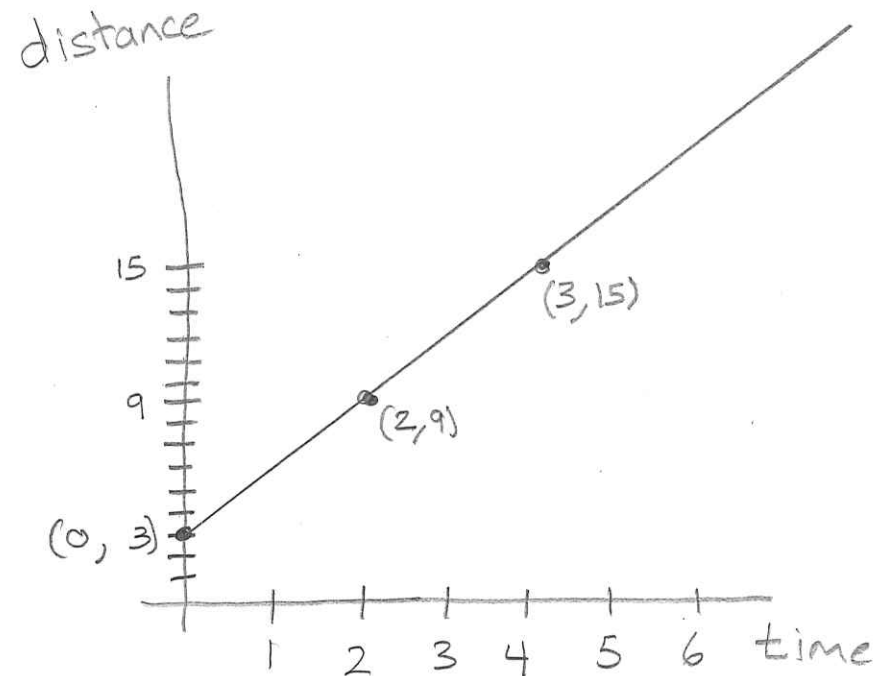
Ron runs at a constant speed. He starts 3 miles from home, and runs 6 miles farther from home every 2 hrs.

- What is his average speed?
- What would the graph of Ron's *distance traveled* look like?

TIME	0 hrs	2	4	6	...
DISTANCE	3 miles	9	15	21	...

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} = \frac{\Delta \text{DIST}}{\Delta \text{TIME}}$$

$$\frac{6 \text{ miles}}{2 \text{ hours}} = \boxed{3 \text{ mph}}$$



NOTE:  $\boxed{\text{SLOPE} = 3}$

Now consider the graph of distance given in the lecture pack.

### Lesson 1 in reading graphs

- Read the description (twice!).
- What are the variables? (Label)
- Make a table of a few values.

TIME (t)	0	5	10	...	70	75
DIST (D)	0	~70	~120	...	~430	~550

(a)  $t=0 \Rightarrow D=0$ ,  $t=5 \Rightarrow D \approx 70$   
 AVERAGE SPEED =  $\frac{70-0}{5-0} = 35$  miles/min

(b)  $t=0 \Rightarrow D=0$ ,  $t=70 \Rightarrow D \approx 430$   
 AVE. SPEED =  $\frac{430}{70} \approx 6.14$  miles/min

(c)  $t=5 \Rightarrow D \approx 70$ ,  $t=10 \Rightarrow D \approx 120$   
 AVE. SPEED =  $\frac{120-70}{10-5} = \frac{50}{5} \approx 10$  mph

(d)  $t=70 \Rightarrow D \approx 430$ ,  $t=75 \Rightarrow D \approx 550$   
 AVE. SPEED =  $\frac{550-430}{75-70} = \frac{120}{5} \approx 24$  mph

**Big Question:** How can we estimate speeds from a distance graph?

**Specific Questions:** What is the average speed from ...

- ...  $t = 0$  to  $t = 10$
- ...  $t = 0$  to  $t = 70$
- ...  $t = 5$  to  $t = 10$
- ...  $t = 70$  to  $t = 75$

AVERAGE TRIP SPEED AT  $t=5$

AVERAGE TRIP SPEED AT  $t=70$

ASIDE

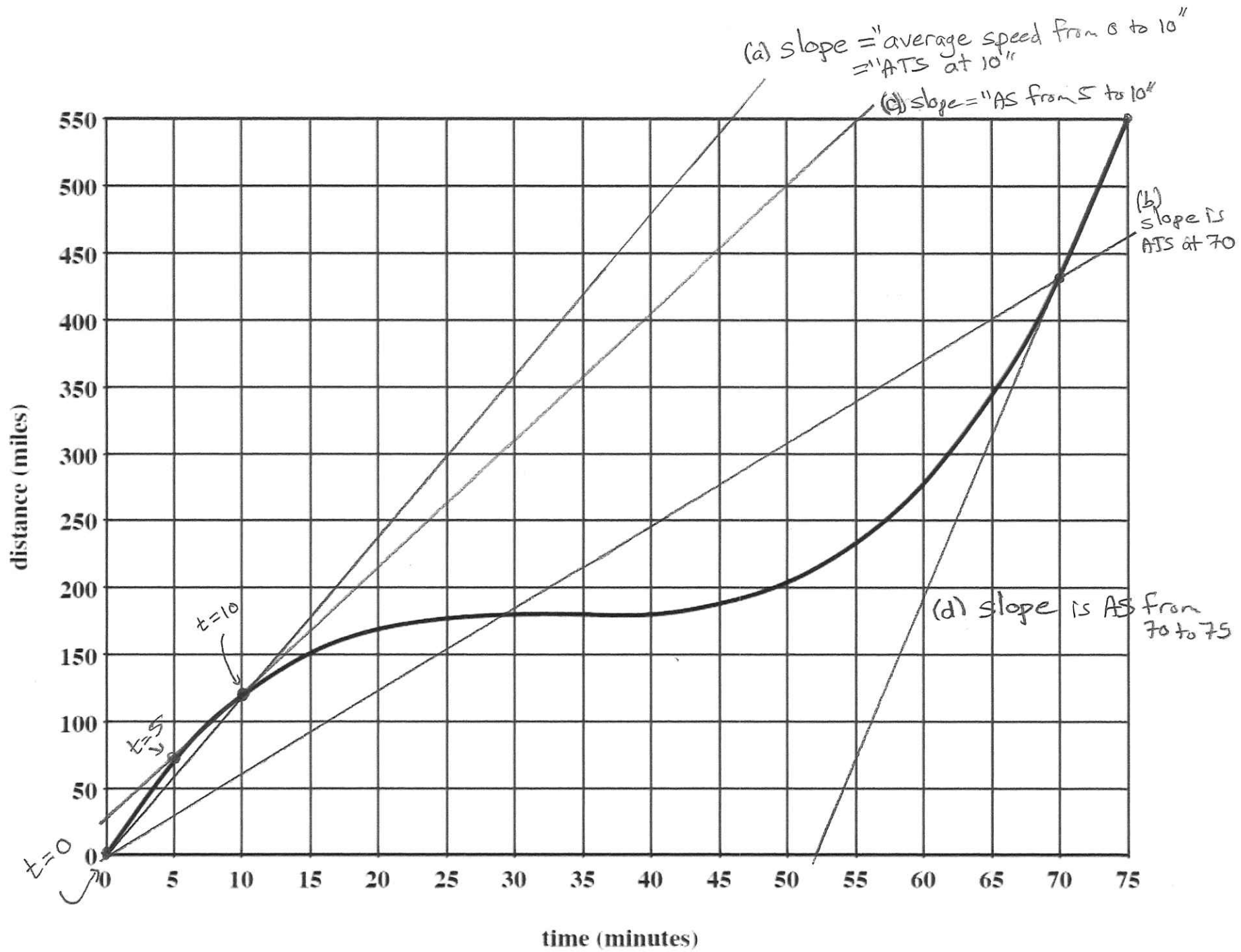
WHEN IS ATS LOWEST?

$t \approx 50$  min

ASIDE

GIVE A 5-MIN INTERVAL WHEN AS IS LOWEST?

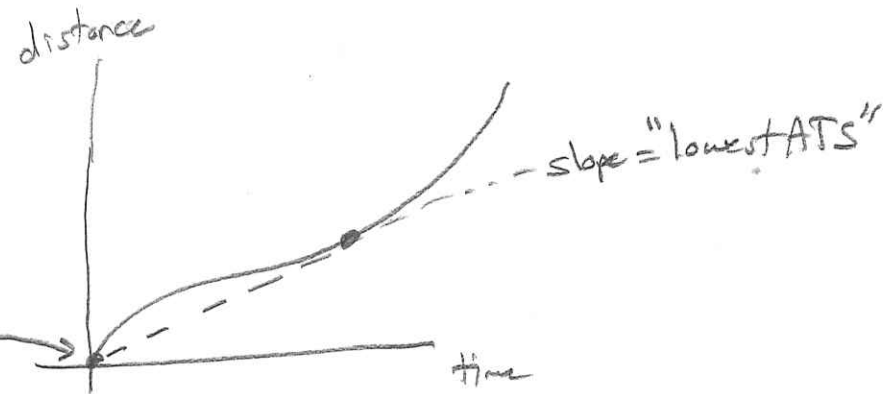
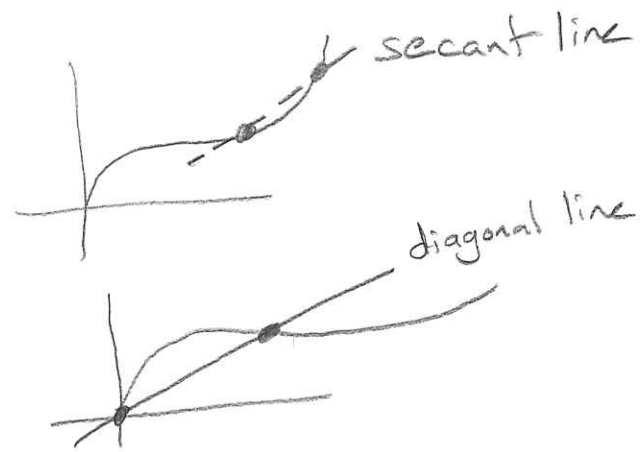
30 to 35



## Graph Terminology and Facts

- A secant line is a line through two points on a curve.
- A diagonal line is a line through the origin.
- Slope of a line =  $\frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$

Pick ANY two points on the same line and you get the same slope!



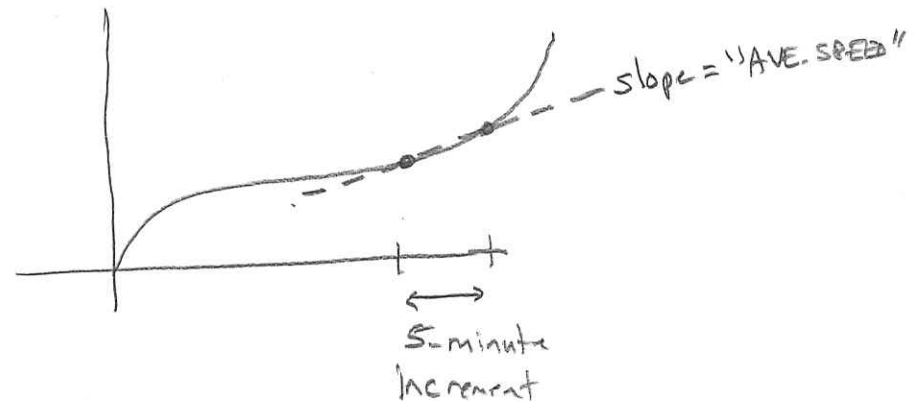
## Summary

### **Overall average rate:**

$$\text{ATS} = \text{average trip speed} = \frac{\text{Total Dist}}{\text{Total Time}}$$

### **Incremental average rate:**

$$\text{AS} = \text{average speed} = \frac{\text{Change in Dist}}{\text{Change in Time}}$$



# Types of Rates

MATH III  
slopes of secants

Average Rates

Instantaneous Rates

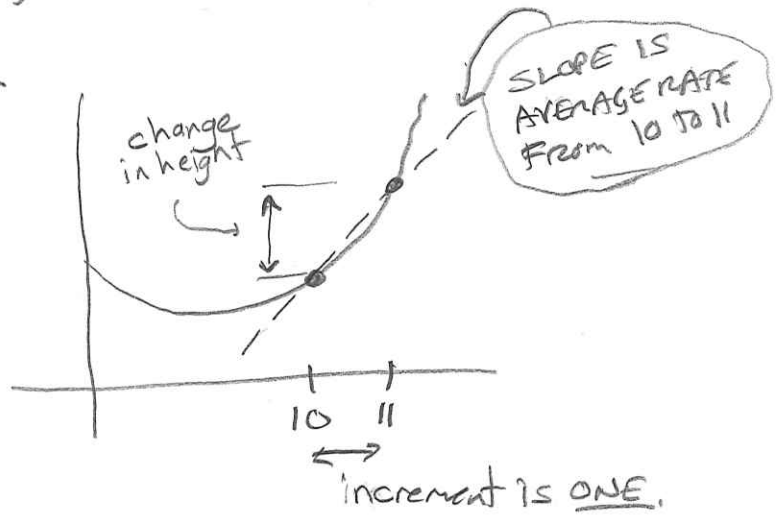
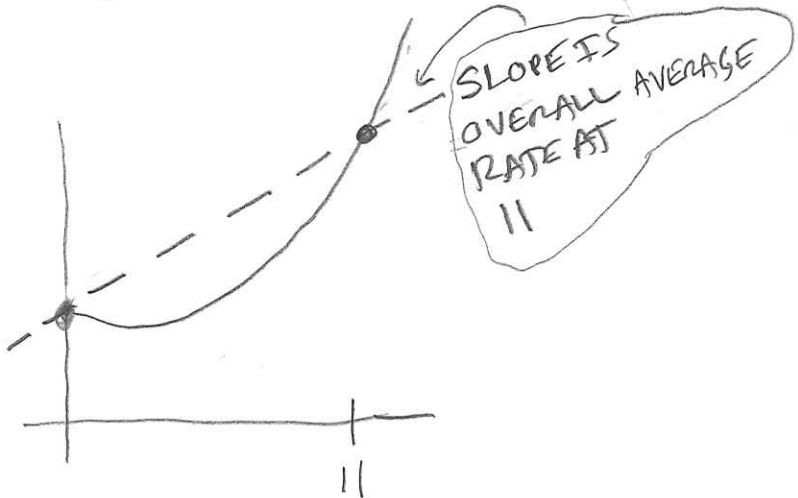
MATH 112  
slopes of tangent lines

Overall

Incremental

BETWEEN TWO GIVEN TIMES

FROM  $t=0$  ON GRAPH TO SOME GIVEN TIME



## *Lesson 2 in reading graphs*

- If given a total distance graph and asked about speed/rates, **get your ruler out** and draw lines.

Use points far apart and easy to read to find the slopes of the lines.

- If given a graph that has increments or rates, **put your ruler away**. Read off values from the data and use a table.

Warm up questions:

1. Find the overall rate of change after 9 minutes?

- DRAW LINE THRU GRAPH AT  $t=0$  and  $t=9$ .
- ESTIMATE TWO POINTS ON LINE  $\begin{cases} (0, 180) \\ (9, 200) \end{cases}$
- COMPUTE SLOPE (RATE).

$$\text{SLOPE} = \frac{200 - 180}{9 - 0} = \frac{20}{9} \approx \boxed{2.22 \frac{\text{°C}}{\text{min}}}$$

**Note: Overall Rate** means starting from wherever the graph starts (not necessarily the origin)

↙ SPEED (RATE)

2. How fast does the temp rise, on average, during the 3-min interval beginning at  $t=4$  min?

$$t=4 \text{ to } t=4+3=7$$

- DRAW LINE THRU GRAPH AT  $t=4$  and  $t=7$
- ESTIMATE TWO POINTS ON LINE  $\begin{cases} (4, 400) \\ (8, 550) \end{cases}$
- COMPUTE SLOPE (RATE)

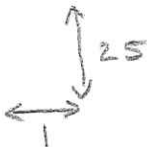
$$\text{SLOPE} = \frac{550 - 400}{8 - 4} = \frac{150}{4} = \boxed{37.5 \frac{\text{°C}}{\text{min}}}$$

How could we answer these?

3. Find a time at which overall rate of change of temp is 25 deg per min.

GIVEN A RATE!  
WORKING BACKWARD!

- DRAW A REFERENCE LINE WITH SLOPE 25.



$(0,0), (1,25), (2,50), (3,75), \dots, (10,250)$

- SLIDE RULER PARALLEL TO THE REFERENCE LINE UNTIL IT IS TOUCHING THE STARTING LOCATION ON GRAPH

- FIND OTHER INTERSECTION

$$t \approx 8.1 \text{ min}$$

SO FROM  $t=0$  TO  $t=8.1$   
THE RATE IS ABOUT  $25 \frac{^{\circ}\text{C}}{\text{min}}$ .

4. Find a two-minute interval during which the incremental rate of change is 10 deg/min.

GIVEN A RATE!

- DRAW A REFERENCE LINE WITH SLOPE 10.



$(0,0), (1,10), (2,20), \dots, (10,100)$

- SLIDE RULER PARALLEL TO THE REFERENCE LINE UNTIL IT IS TOUCHING THE GRAPH AT TWO LOCATIONS WHICH ARE 2-MIN APART.

- TWO ANSWERS!

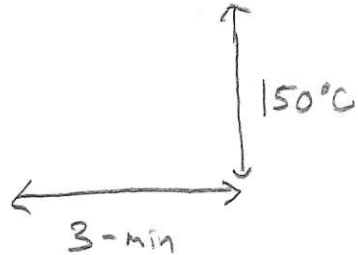
$$t \approx 4.8 \text{ TO } t \approx 6.8 \text{ min}$$

$$t \approx 9.1 \text{ TO } t \approx 11.1 \text{ min}$$



5. Find a three-minute interval during which the temp rises by 150 deg.

WANTS



SO GIVEN RATE =  $\frac{150^{\circ}\text{C}}{3 \text{ min}} = 50 \frac{\text{C}}{\text{min}}$  & WANT A 3-MINUTE INTERVAL

- DRAW REFERENCE LINE WITH SLOPE 50  $(0,0), (1,50), (2,100), (3,150) \dots$
- SLIDE RULER PARALLEL TO THE REFERENCE LINE UNTIL IT IS TOUCHING THE GRAPH AT TWO LOCATIONS WHICH ARE 3-MIN APART.

TWO ANSWERS

- $t \approx 0.9$  to  $t \approx 3.9$  min
- $t \approx 3.7$  to  $t \approx 6.7$  min

## Supplement 3-4 (Reference Lines)

Temp vs. time for a chemical reaction.

